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Diquark stars with extended scalar diquarks and their stability

S.K. Karn, R.S. Kaushal[a], Y.K. Mathur[b]

Department of Physics and Astrophysics, University of Delhi, Delhi-110 007, India

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Abstract. Within the framework of an effective ϕ^4-theory, an attempt is made to study diquark stars and their stability with extended scalar diquarks (ESD). In this context, an equation of state (EOS) for the ESD gas is obtained. We find the EOS for the ESD gas to be stiffer than that for a point-like diquark and/or quark gas. This EOS is then used to investigate various properties of the diquark stars. In particular, the mass and radius of the maximum mass star with ESD matter turn out to be larger than those obtained with point-like diquark and/or quark matter. However, they are in conformity with the predictions available for soliton and boson stars. The stability of ESD stars against radial oscillations is also investigated.

1 Introduction

It is now well known that from the point of view of studying hadronic systems at the level of energy density of $2\,\mathrm{GeV/fm^3}$ (i.e., greater than the standard nuclear matter density) and/or at a temperature of about 200 MeV, there exists compelling evidence [1] to describe them in terms of their constituents, the quarks and gluons. This is so because the interactions among quarks and gluons at such high densities become weak at short distances mainly due to the colour screening effect [2]. It is thus likely that the systems undergo a transition from hadronic to a new phase of strongly interacting coloured quarks and gluons in their unconfined state which is called "quark–gluon plasma" (QGP) and the experimental research into this new state of matter is the central goal of the present and planned relativistic heavy ion collisions (RHIC) being carried out at CERN and at BNL. As a result, there has been considerable interest in recent years to study the RHIC and subsequently the QGP. Due to the fact that in a QGP phase the quarks and gluons may not be non-interacting, quarks might pair up non-perturbatively and create diquarks, mainly due to spin–spin interactions. There may be intermediate phases (may be even for a shorter duration), namely the diquark–quark gluon phase and the diquark–gluon phase, between the two extreme regimes of hadron and quark phases. Moreover, the importance of quark pairing, particularly in the high-density regime, has also been emphasized earlier (see, for example, [3]). It is in this context that the role of diquarks becomes of the utmost importance. In fact, the possibility of the existence of these intermediate phases was first pointed out and explored by Ekelin and Frederiksson [3] and later on emphasised by others [4–8].

In our recent works [8,9] (from now on referred to as KKM), important theoretical support to the idea that a QGP under certain thermodynamic conditions may contain a significant fracton of extended scalar diquarks (ESD) has been put forward. It is also noticed that ESD matter is energetically more favoured not only as compared to point-like scalar diquark matter but also to quark matter. No doubt the treatment of diquarks as point-like objects is a mathematically idealised situation, but barring a few exceptions of some investigations of point-like diquarks, the study of the role of diquarks has not yet been carried out in full detail, especially considering that the diquarks may turn out to be extended objects.

The purpose of this paper is twofold: Firstly to study diquark stars and their stability with ESD matter and secondly to highlight its possible effects in the context of stellar studies. For this purpose, we use an effective ϕ^4-theory along the lines of Donoghue and Sateesh (DS) [4], and extend our earlier work [8] with a view to studying diquark stars and their stability with ESD matter. In particular, a model for the diquark [10], accounting for its size, and in conformity with several ground state properties of baryons, is employed in the present work. The basic assumption in using these phenomenological models [4,8] is that the bound diquarks in nucleons retain the same properties in a medium such as QCD plasma. The plan of the paper is as follows. Since it is of relevance here, we briefly review the role of quark degrees of freedom in understanding the early universe in Sect. 2, and we highlight the role of diquarks in star formation in Sect. 3. We then deduce an equation of state for the ESD gas within the framework of the ϕ^4-theory in Sect. 4. Section 5 is devoted to the study of ESD stars by solving the Oppenheimer–Volkoff equations. The results are discussed in Sect. 6, and

[a] UGC Research Scientist: e-mail: rkaushal@physics.du.ac.in
[b] e-mail: Yogesh@DUCOS.ernet.in

aspects of the stability of ESD stars are investigated in Sect. 7. Finally, concluding remarks are made in Sect. 8.

2 A brief review of the understanding of the early universe and quark degrees of freedom

There is a strong belief that the universe mainly consisted of a very compact (superdense) matter [11] in the early stages of its history. Such high densities were treated before as irrelevant for the normal range of physics, but later on, in order to explain several astrophysical and cosmological situations (i.e., the centre of a neutron star, supernova explosions or black holes etc.), the role of such high densities became of vital importance. During the 1940s there were speculations that the central densities of some stellar objects might be higher than those of nuclear matter. Oppenheimer and Volkoff [12] accounted for the relativistic effects to understand the massive, superdense neutron cores. Later, quark degrees of freedom at such densities have been emphasised to exist [13] and were found to play an important role in the formation of quark stars. Collins and Perry [2] have extensively discussed the picture of the three quark structure of baryons and its importance for the understanding of the superdense matter expected to be present in several stellar objects such as in the cores of neutron stars. They have strongly suggested a possible phase transition from neutron matter to a uniform quark matter at such high densities. Since then many authors have considered quark matter [1,19] and have discussed [14–19] various properties of quark stars and/or neutron stars with a core of quark matter. In spite of all these developments over the years, it appears impossible [15] to predict with confidence the interior constitution of neutron stars. As a matter of fact, the physical behaviour of matter under such extreme densities has not been completely understood so far. Also, the effects that are of minor importance at nuclear densities may be of major importance at such high densities, and the physical quantities describing [20] the interior as well as the surface of the neutron star (or for that matter of any stellar object) depends highly on the possible states of the dense matter in its core.

From the point of view of a field theoretic approach there have been speculations that the scalar fields not only play [21,22] a critical role in the evolution of the early universe but also may explain [23] some important cosmological problems, like the problem of the cosmological missing mass. It is worthwhile mentioning here that of all possible states, scalar diquarks (i.e., the colour antitriplet, spin zero diquarks) have been found to be [24] energetically more favoured. It is strongly suggested [25] that the states of scalar diquarks play important roles for the explosive phase, and subsequently their condensation in a QGP phase is responsible for the ultimate collapse of the infalling stellar matter. The states of scalar diquarks in a QCD plasma are also expected [25] to play an important role in the core of a would-be supernova. This is put forward because neither nucleons at about 1 fm apart nor the phase transition to a QGP are found to be violent enough to affect the explosion which is to take place.

In another context, in the literature "quark pairing" is considered [3] analogously to the formation of Cooper pairs in superconductors. Quarks are said [25] to show an even "un-natural pairing up" to form diquarks. Such bindings are possible in the presence of repulsive force between the quarks if some of the forces are more repulsive than others. No doubt it appears that such an analogy would suggest weak binding forces among the quarks, but the latter also have the colour degrees of freedom besides having formed a bound state (diquark). In the present work, we consider the regime just above the deconfinement where the forces between the quarks are expected to be fairly strong – to the extent that a large fraction or all the quarks pair up to give rise to a bound state, thereby improving upon the EOS. Once the boson system of such paired but extended objects is formed, studies in these directions become of considerable interest (see, for example, [3] and references therein). However, here we shall look for the possible role of ESD in the form of matter and also in the context of stars with that matter in the following sections.

3 Role of diquarks in star formation

DS [4] have explored the possibilities of diquark cluster formation in a QGP in the density regime higher than that required for the deconfinement. They treat the quarks at such high densities as a free Fermi gas and take into account the spin–spin interactions [26] among them. They then transform the quark interactions into an effective ϕ^4-theory for the scalar diquarks thus formed. In fact, the theory describes the diquarks as self-interacting bosons by fixing the coupling constant λ in the Lagrangian for a colour triplet field by using the ϕ^4-theory. They fix the mass of the scalar diquark by the N–Δ mass difference and calculate the energy dependence of the interacting scalar diquark gas as a function of density by using a Gaussian momentum distribution for the scalar diquark gas. Finally, they arrive at the very significant conclusion that the energy of the scalar diquark gas at the density range above deconfinement is considerably lower than that of the quark gas, and at still higher densities the diquarks break up into quarks. Thus a quark phase occurs again.

Kastor and Traschen (KT) [6], using the model of DS, discuss the astrophysical realisation of the diquark matter and also of a neutron star with that matter in its core. They calculate the pressure of a diquark gas from the diquark energy obtained by DS, describe the formation of diquark stars, and deduce various other properties of neutron stars. They consider a mixture of quarks and diquarks surrounded with and without a low density neutron envelope. Treating the core as consisting of diquark matter or as a mixture of quark and diquark matter in different proportions, they reproduce several features of neutron stars. For an isoscalar mixture of quarks (equal number of up and down quarks) they find that the fraction of quark pairs drops to half at about eight times the nuclear matter density. However, for a zero charge mixture of up and down quarks coming from pure neutron matter, they find that the fraction of quark pairs drops to two thirds only at ten times the nuclear density. Horvath et al. [7] also have considered the model of DS and discussed the possibility of having a self-bound, stable state of the diquark matter in abundance in stellar objects. Consequently, the diquark matter would play an important role or rather a dominant role in a variety of compact energetic astrophysical systems such as neutron stars.

4 Equation of state for ESD matter

Following KKM [8], the distribution of ESD in a QGP in terms of a modified Gaussian distribution function $F(k)$ of diquarks is of the form

$$F(k) = \left[N/2(2\pi a^2 m_\mathrm{D}^2)^{3/2}\right] ((k/b)^2 + 1)^{-2} \times \exp(-k^2/2a^2 m_\mathrm{D}^2) \,, \quad (1)$$

where N is the number of quarks, a is the Gaussian width parameter, m_D is the mass of the diquark, and $b = (2/B_\mathrm{oc})$ with B_oc as the first Bohr radius [10]. The $F(k)$ is normalised [4,28] as $\int d^3k F(k) = N/2$ and the total energy of the ESD gas turns out to be

$$E_\mathrm{D} = I_1 + (\lambda/2V) I_2^2 \,, \quad (2)$$

where the momentum space integral $I_1 = \int d^3k (k^2 + m_\mathrm{D}^2)^{1/2} F(k)$. The integral I_2 is obtained by multiplying the integrand I_1 by $(k^2 + m_\mathrm{D}^2)^{-1}$. In terms of the quark number density $\rho (\equiv N/V)$, where V is the volume occupied by the quarks, the energy of ESD gas can be expressed as

$$E_\mathrm{D} = \frac{N m_\mathrm{D}}{4\pi} \left[2\sqrt{2\pi}\, a^{-a} \mathscr{I}_1 + \lambda\, a^{-6} (\rho/m_\mathrm{D}^3) \mathscr{I}_2^2\right] \,, \quad (3)$$

where $\mathscr{I}_1 = \int_0^\infty dk' k'^2 \; (k'^2 + 1)^{1/2} \tilde{g}(k')$ with $\tilde{g}(k') = g(k') \exp(-k'^2/2a^2)$, and $g(k') = ((m_\mathrm{D} k'/b)^2 + 1)^{-2}$. The integral $\mathscr{I}_2$ is obtained by multiplying the integrand of $\mathscr{I}_1$ by $(k'^2 + 1)^{-1}$. Note that the variable k of (1) is replaced by $(m_\mathrm{D} k')$ in (3) for dimensional considerations.

The expression for the pressure for the ESD gas is calculated by using the relation $p = -(\partial E_\mathrm{D}/\partial V)_N$ and it turns out to be

$$p = (I_2(3 - 2I_4)(\lambda/2) + V I_3) / 3V^2 \,, \quad (4)$$

where I_2 is the same as in (2) but the integrals I_3 and I_4 are now obtained by multiplying the integrand of I_1 by $k^2(k^2 + m_\mathrm{D}^2)^{-1}$ and $k^2(k^2 + m_\mathrm{D}^2)^{-2}$, respectively.

As far as the expression (4) for the pressure of the ESD gas is concerned: it can be expressed in terms of the matter density ρ_m as

$$p = \alpha \rho_\mathrm{D} + \beta \rho_\mathrm{D}^2 \,. \quad (5)$$

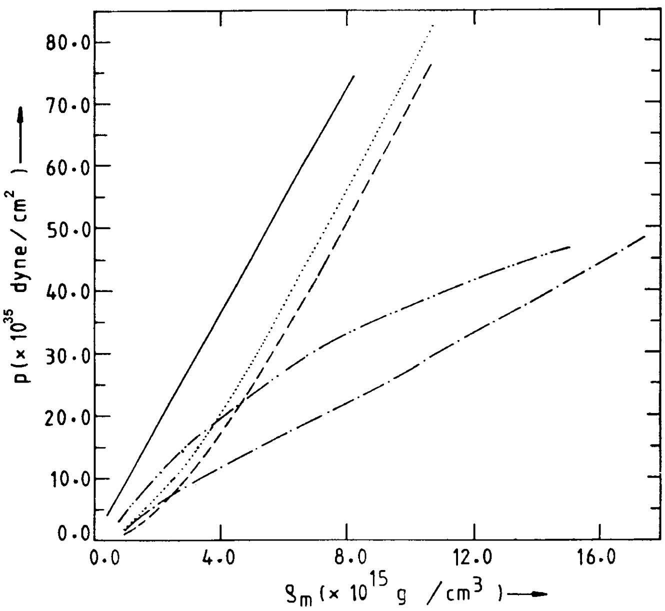

Fig. 1. The equation of state for ESD matter (continuous curve). Results corresponding to neutron matter (double dot-dashed curve) with spin-2 meson coupling constant $f^2 = 0.6$ and QCD derived quark matter (dot-dashed curve) are shown for comparison from Anand et al. [16]. Results for neutron matter of Pandharipande and Smith [20] (dotted curve) and of Malone et al. [20] (dashed curve) are also shown for comparison

Here $\rho_\mathrm{D} (\equiv m_\mathrm{D} \cdot N/2V)$ is the diquark matter density, $\alpha = x_1 \mathscr{I}_2'$ and $\beta = 1.5\, x_1 \lambda \mathscr{I}_1' (3\mathscr{I}_1' - 2\mathscr{I}_3')$ with $x_1 = (1/6\sqrt{2\pi}) a^{-3} m_\mathrm{D}^{-4}$, $\mathscr{I}_1' = \int_0^\infty \mathrm{d}k \; k^2 (k^2 + m_\mathrm{D}^2)^{-1/2} \; \tilde{g}'(k)$, $\tilde{g}'(k) = g'(k) \exp(-k^2/2m_\mathrm{D}^2 a^2)$, $g'(k) = ((k/b)^2 + 1)^{-2}$ and the integrals $\mathscr{I}_2'$ and $\mathscr{I}_3'$ are obtained by multiplying the integrand $\mathscr{I}_1'$ by k^2 and $k^2(k^2 + m_\mathrm{D}^2)^{-1}$, respectively. For the ESD gas the pressure p as a function of matter density ρ_m is shown in Fig. 1. It should be noted that in this figure (and for that matter in all subsequent figures) the following conversion factors are used: $1\,\mathrm{dyne/cm^2} = 4.793 \times 10^{-27}\,\mathrm{MeV^4} = 5.586 \times 10^{-40}\,\mathrm{M_\odot/km^3}$ and $1\,\mathrm{g/cm^3} = 4.314 \times 10^{-6}\,\mathrm{MeV^4} = 5.028 \times 10^{-19}\,\mathrm{M_\odot/km^3}$.

Regarding the relative magnitude of the terms in the equation of state (EOS) (5) note that the ratio β/α turns out to be of the order of 10^{-5}, implying a very small contribution of the second term in (5). For this reason the p vs. ρ_m curve in Fig. 1 for the ESD matter turns out to be nearly a straight line. As far as the study of the EOS for stellar matter in the presence of quark degrees of freedom is concerned several studies have been carried out [14–17,19] in the literature. For example, for the sake of comparison the results of Anand et al. [16] are shown in Fig. 1. The dot-dashed curve corresponds to the results for the quark matter and the dashed curve is for the neutron matter with a spin-2 meson coupling constant $f^2 = 0.6$. In this figure, the classic results of Pandharipande and Smith [20] and of Malone et al. [20] for neutron matter are also shown for comparison. In addition, the numerical values for the EOS are also displayed in Table 1 mainly for the sake of a more precise comparison of results for the quark and neutron matter at a later stage. It should be emphasised here that the present EOS allows only for a limited range of p and ρ_D in the case of ESD gas. However, this window is somewhat wider than the one obtained [6] for the point-like diquarks.

5 Extended diquark stars: calculational details

Having thus obtained the EOS (5) for the ESD matter in the preceding section we now proceed to study the structure and hydrostatic equilibrium of ESD stars using the well-known Tolman–Oppenheimer–Volkoff [27,12] (TOV) equations, namely

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)}{r^2}\left[\rho_\mathrm{m}(r) + p(r)\right]\left[1 + 4\pi r^3 p(r)/M(r)\right] \cdot \left[1 - \frac{2GM(r)}{r}\right]^{-1} , \quad (6)$$

$$\frac{\mathrm{d}M(r)}{\mathrm{d}r} = 4\pi r^2 \rho_\mathrm{m}(r) , \quad (7)$$

where the various symbols have their usual meanings. For this purpose, we use the EOS (5) and integrate the coupled equations (6) and (7) numerically. In order to handle these equations the boundary conditions used here are that the centre be free from singularities and the pressure or matter density at the surface is fixed such that

$$M(0) = 0 ; \quad p(r = R) = p\left(\rho_\mathrm{m} = \rho_\mathrm{m,surface}\right) , \quad (8)$$

where R is the radius of the configuration. Here, $\rho_\mathrm{m,surface}$ is taken as the cut-off density, which in the present case is the allowed minimum value of ρ_m for the ESD formation and the same is obtained by minimising the energy of the ESD gas, E_D, from (3) with respect to the Gaussian width parameter a. For the details of the calculations of this part we refer to our earlier works [8,28]. However, to deduce the properties of the ESD stars the computation of the TOV equations (6) and (7) is carried out keeping in mind the boundary conditions (8) and using the EOS (5) in the numeric form. For this purpose, we use the following prescription.

(i) The central density (ρ_c) and some initial value of the radius r (say $r = r_0 = 0.01\,\mathrm{km}$) and accordingly the mass from ($4\pi r_\mathrm{c}^3 \rho_\mathrm{c}/3$) are used as input for ρ_m, r and M, respectively. Note that ρ_c is taken from the allowed range of ρ_m for ESD formation in QGP, and the TOV equations (6) and (7) are numerically solved simultaneously for $m(r)$ and $p(r)$. Corresponding to this value of ρ_m, p from the EOS is also used as input.

(ii) While M so obtained becomes the input for the second round, a search for ρ_m from the EOS is made corresponding to the output value of p using either the numeric EOS or by using the standard interpolation formula (see [20])

$$[(\ln p - \ln p_j)/(\ln \rho_\mathrm{m} - \ln \rho_j)] = (\ln p_{j+1} - \ln p_j)/(\ln \rho_{j+1} - \ln \rho_j) , \quad (9)$$

where p_j and p_{j+1} are the values of the pressure between which the output value p lies, ρ_j and ρ_{j+1} are the corresponding values of ρ_m in the EOS. Note that the formula (9) is helpful when the output value of p does not match with a value of p in the numeric EOS. In fact, this normally is the case. The value of ρ_m so obtained along with the value of p now becomes the input for the next round which starts at a newer value of the radius.

(iii) The above process is repeated until all the values of p and ρ_m in the numeric EOS are covered in accordance with the second boundary conditions in (8). In this way, one obtains the properties of the configuration corresponding to a given input value of ρ_c.

(iv) A set of numerical results is obtained for the allowed values of ρ_m by treating ρ_m as ρ_c in the input. Finally, a maximum mass configuratoin is searched from among the results for all values of ρ_c.

6 Results and discussion

The results of the calculations are shown in Figs. 1–4. In these figures the continuous curves indicate the results of the present calculations for the ESD case. A plot of the EOS for the ESD gas, which has already been described, is depicted in Fig. 1. In order to determine the maximum mass of ESD stars, the configuration mass as a function of the radius R and the central density ρ_c is plotted in Figs. 2 and 3, respectively. For comparison, results obtained by other authors based on different models, namely, those of KT [6] (dashed curves) for a zero charge mixture of diquarks and quarks with (curve a) and without (curve b) a low density neutron envelope, of Ghosh and Sahu [17] for the bag model (dotted curve) and the two flavour chiral dielectric quark model without gluon interactions (dot-dahsed curve), and of Chandra and Goyal [18] (double dot-dashed curve in Fig. 2 and dashed curve in Fig. 4) for soliton stars, are also displayed along with our results. The variation of the configuration mass as a function of the central density is shown in Fig. 3 for all the cases discussed in Fig. 2 except for the case of a soliton star of Chandra and Goyal [18]. However, with regard to the results of other authors the abscissa is scaled down by a factor of ten and the ordinate is scaled up by a factor of 4 in Fig. 3. It is to be mentioned here that in Fig. 2 of all the curves the trend of curve (b) appears to be different in the sense that the overall mass of the star decreases with the increase in its radius for a zero charge mixture of diquarks and quarks even without a low-density neutron envelope.

Density profiles for the ESD stars for some typical values of ρ_c which are used as input in the computation of the TOV equations (6) and (7) are shown in Fig. 4. In this figure, curves (a), (b), (c) and (d) correspond to the values of ρ_c of 81.96, 2.92, 1.49 and 0.5 (in units of $10^{14}\,\mathrm{g/cm^3}$), respectively. Note that the curve (c) here corresponds to the maximum mass $8.92\,M_\odot$ obtained for the ESD stars (cf. Fig. 2). The corresponding radius in our model turns out to be 50.7 km. The density profiles obtained for the soliton star [18] corresponding to the maximum mass (dashed curve) is also shown for comparison. It is interesting to

Table 1. Equation of state for ESD matter and its comparison with the results of other authors obtained for neutron and quark matter (ρ and p are in units of 10^{14} g/cm^3 and 10^{35} dyne/cm^2, respectively)

S. NO.	Present calculations (ESD matter)		Pandharipande and Smith [20] [neutron (solid model) matter]		Malone et al. [20] [neutron (model V N) matter]		Anand et al. [16] (quark matter)	
	ρ	p	ρ	p	ρ	p	ρ	p
1.	0.1345	0.1235	1.890	0.0634	1.700	0.0119	2.606	0.3487
2.	0.2491	0.2287	2.42	0.0876	2.55	0.0293	19.76	4.996
3.	0.2947	0.2706	3.57	0.298	3.42	0.060	68.91	18.77
4.	0.3713	0.3409	4.97	0.614	4.31	0.109	176.32	49.38
5.	0.4969	0.4561	6.04	0.926	5.20	0.183	375.8	106.8
6.	0.8441	0.7748	6.83	1.18	7.04	0.409	709.6	203.5
7.	0.9431	0.8656	8.75	1.18	8.95	0.761	1227.0	354.1
8.	1.155	1.060	10.7	1.86	10.9	1.26	1987.0	575.7
9.	1.488	1.366	14.3	3.327	13.1	1.99	3054.0	887.8
10.	2.921	2.678	17.8	4.98	15.3	2.85	4503.0	1312.0
11.	3.836	3.516	26.7	10.6	17.6	3.71	6415.0	1873.0
12.	5.814	5.325	35.7	17.0	20.1	4.92	8881.0	2598.0
13.	6.039	5.531	53.5	32.0	22.6	6.23		
14.	8.230	7.532	71.3	48.2	27.0	8.58		
15.	14.06	12.85	107.0	82.3	31.6	11.4		
16.	19.59	17.89	143.0	117.0	41.9	18.5		
17.	25.72	23.46	178.0	153.0	53.5	27.6		
18.	50.50	45.96	196.0	171.0	77.0	48.3		
19.	81.97	74.48	–	–	106.0	76.2		

note that not only the trend, but also the magnitudes of the densities for ESD stars are in agreement with the predictions available for the soliton stars [29]. For small values of ρ_c the behaviour of the density profiles becomes flat as is the case with the predictions of other authors, particularly for the soliton stars [18].

We emphasise here that the trend and the behaviour of the results for the ESD stars, by and large, are the same as the predictions of other models for quark and/or diquark stars as well as soliton stars, except for the fact that the magnitudes for M and R in our case turn out to be large. However, as mentioned above, these figures are in agreement with the predictions made for soliton and boson stars [29,30]. In fact, the prediction of maximum mass star of mass 20 $M_\odot$ and radius 60 km with the soliton model has been made by Cottingham and Mau [29] by introducing a temperature dependence in the Lee–Wick model [30]. It may also be mentioned that in a way the solitonic character, to some extent, is built in [35] the ϕ^4-theory used in our case. It can further be argued that the large mass and radius for the ESD stars obtained in our model can be attributed to the mutual interactions among scalar diquarks because scalar fields perhaps are capable of explaining the missing mass of the universe and can play a critical role [21,23] in the evolution of the early universe.

7 Stability of ESD stars

The stability of stellar objects against their constituents and the corresponding interactions has been the subject of study for a long time [31,32]. A theory of stability accommodating these features a priori can be examined either in terms of oscillations of the radial coordinate [32, 33] and/or be assumed to be due to the occurrence of the phase transition(s) (in the present case it may be either from the quark–gluon phase to the diquark–gluon phase or from the diquark–gluon phase to the hadronic phase). While the former situation provides a necessary condition for the stability of the stellar objects in terms of their gravitational mass M and central density ρ_c through

$$\frac{\mathrm{d}M}{\mathrm{d}\rho_c} > 0\,, \tag{10}$$

the latter, on the other hand, can possibly offer a sufficiency condition in this regard. Condition (10) suggests [32] the value of the adiabatic index $\bar{\Gamma}_1 > 4/3$, which, after accounting for the post-Newtonian approximation, becomes [32]

$$\bar{\Gamma}_1 > (4/3) + 2GM\kappa/(RC^2)\,, \tag{11}$$

where $\kappa \sim 1$ and depends on the structure of the star. Note that two out of the three typical values of ρ_c listed in Table 3 conform to condition (10) as is clear from Fig. 3.

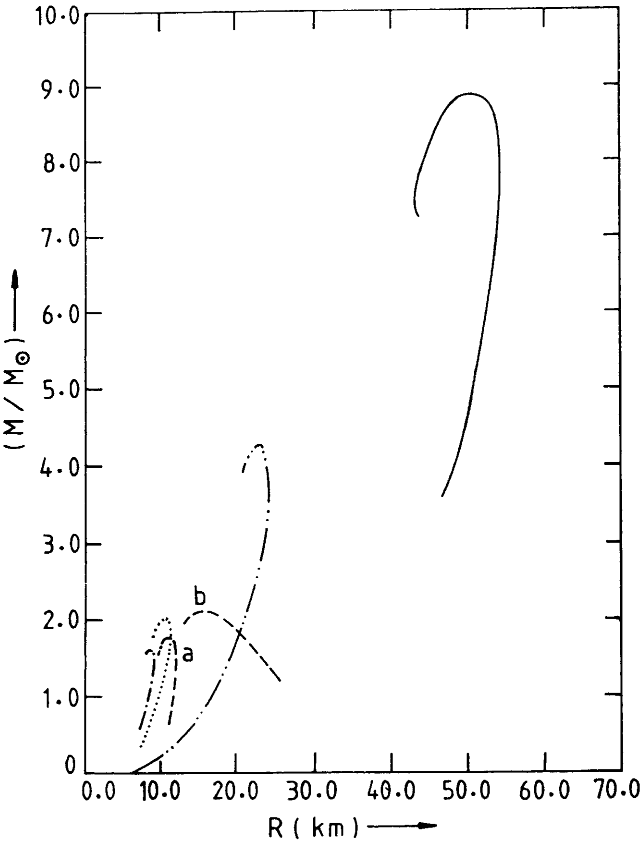

Fig. 2. Maximum mass configuration as a function of the radius. Dashed curves **a** and **b** are the results of KT [6] for a zero charge mixture of diquarks and quarks with and without a low-density neutron envelope, respectively. Dotted and dot-dashed curves are from Ghosh and Sahu [17]. The double dot-dashed curve is from Chandra and Goyal [18]. The continuous curve represents the results of the present ESD model

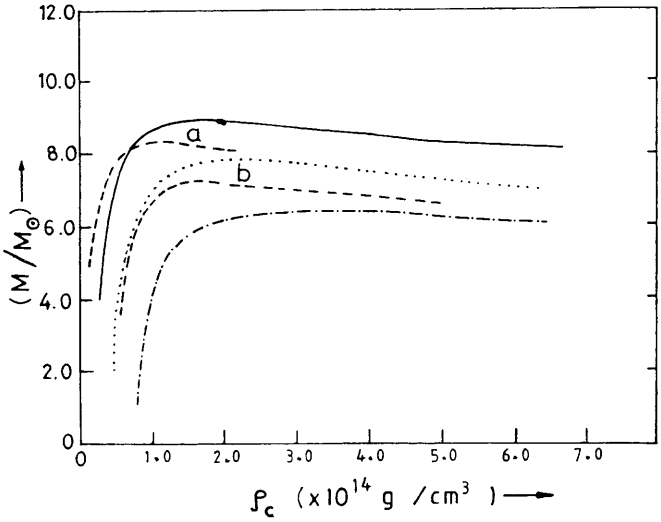

Fig. 3. Plot for mass versus central density. Dashed curves represent the results of KT [6] and dotted and dot-dashed curves are from Ghosh and Sahu [17]. Continuous curves are the results of the present ESD model. Note that the abscissa is scaled down by a factor of ten and the ordinate is scaled up by a factor of four with regard to the results of other authors

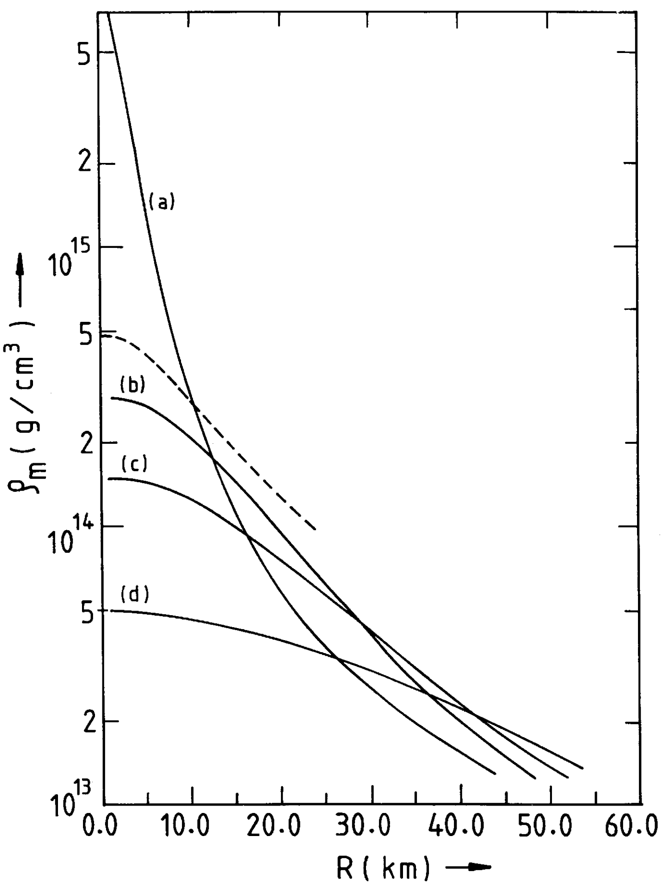

Fig. 4. Density profile for diquark and soliton stars. The dashed curve represents the results of Chandra and Goyal [18] for the soliton star based on the Lee–Wick model and continuous curves represent the results of the present ESD model

Another aspect through which the stability of the ESD stars can be looked into, is in terms of microscopic collapse [34] studied through the inequalities

$$p \geq 0 , \quad \text{and} \quad \frac{\mathrm{d}p}{\mathrm{d}\rho} \geq 0 , \tag{12}$$

where $(\mathrm{d}p/\mathrm{d}\rho)^{1/2}$ is a measure of the hydrodynamic phase velocity of sound waves in the stellar matter under consideration. The present results while conforming to these inequalities (cf. Fig. 1 and Table 1), however, provide somewhat larger values of the speed of sound c_s given by $c_s = (\mathrm{d}p/\mathrm{d}\rho)^{1/2}$. Present results for c_s, along with the ones so obtained by other atuhors for the neutron and quark matter are shown in Table 2. In this regard, it may be mentioned that while there exists a controversy on the validity of the inequality $c_s^2 = (\mathrm{d}p/\mathrm{d}\rho) \leq c^2$ in the literature (see, e.g. [34]), the causality through this inequality has limited the value of the mass corresponding to the maximum mass configuration to $3M_\odot$ (see, e.g., Ellis et al. [14] and Kalogera and Baym [34]) which is much smaller than the one obtained in the present work or in the works pertain-

Table 2. Velocity of sound, $c_s = (dp/d\rho)^{1/2}$, in the stellar matter (in units of 10^{10} cm/s) as obtained from various equations of state in the different density ranges

S. No.	Density range (10^{15} g/cm^3)	$c_s = (dp/d\rho)^{1/2}$			
		Present	Pandharipande and Smith [20]	Malone et al. [20]	Anand et al. [16]
1.	$\rho < 5.0$	2.999	2.287	2.154	1.646
2.	$5.0 < \rho \leq 8.0$	2.943	3.049	1.355	2.588
3.	$8.0 < \rho \leq 12.0$	3.906	3.410	3.102	–

ing to soliton and boson stars. For further details on this and related aspects, we refer to the work of Karn [9].

In the present work, we have also computed the right hand side of inequality (11) for different mass–radius configurations of ESD stars, obtained by solving the TOV equations (6) and (7). It is found that the maximum value obtained for this quantity corresponds to the star having $\rho_c = 3.84 \times 10^{14}$ g/cm^3 which leads to $M = 8.57\,M_\odot$ and $R = 46.5$ km. Interestingly, for values of ρ_c less than this, our results show a decrease in the values of the correction term $2GM\kappa/(RC^2)$ whereas for values of ρ_c higher than this the correction term attains almost a constant value which is approximately the same as the one obtained by Kettner et al. [32] for a charm star corresponding to mass and radius $\sim 1.3\,M_\odot$ and ~ 8 km, respectively. However, the trend of this correction term obtained here is just the opposite to the one reported by Haensel et al. [33] for the strange quark stars.

The pressure averaged value of the adiabatic index, $\bar{\Gamma}_1$, is also calculated in the present model by using the formula [32]

$$\bar{\Gamma}_1 = \left[\int_0^R \Gamma_1(r)p(r)r^2 dr\right] \Big/ \left[\int_0^R p(r)r^2 dr\right], \qquad (11')$$

for three typical values of ρ_c, namely 0.497, 1.488 and 2.921 in units of 10^{14} g/cm^3, where the middle value corresponds to the maximum mass configuration of ESD stars, namely $M = 8.9\,M_\odot$ and $R = 50.7$ km. The behaviour of $\Gamma_1(r)$ as a function of $\rho(r)$ is investigated and it is found [9] that the window of $\Gamma_1(r)$ becomes broader as one proceeds from the higher values of ρ_c to the lower ones. In other words, for an ESD star with higher values of ρ_c, $\Gamma_1(r)$ starts from smaller values and attains nearly a constant value with the decrease of $\rho(r)$ as compared to the stars with smaller values of ρ_c. The values of $\bar{\Gamma}_1$ obtained from (11′) turn out to be small as compared to those obtained from the right hand side of inequality (11) as shown in Table 3.

Table 3. Results for $\bar{\Gamma}_1$ corresponding to three typical values of ρ_c

S. No.	ρ_c(10^{14} g/cm^3)	$\bar{\Gamma}_1$ (from (11′)	$(4/3) + 2\kappa GM/(RC^2)$ (cf. (11))
1.	0.497	0.99989	1.7362
2.	1.488	0.99987	1.8524
3.	2.921	0.99991	1.8750

8 Summary and conclusions

We have studied diquark stars with ESD matter, within the framework of an effective ϕ^4-theory, and their stability. In this context, an equation of state for an ESD gas is obtained and various properties of ESD stars, namely the M vs. R curve, the M vs. ρ_c curve and the behaviour of the density profiles are studied. For the ESD stars, it is found that the maximum mass which such a star can attain is $8.92\,M_\odot$ and the radius is 50.7 km corresponding to the central density 1.488×10^{14} g/cm^3. The magnitudes of M and R for a maximum mass star with ESD matter turn out to be larger than those obtained with point-like diquarks and/or quarks. However, they are in agreement with the predictions made for boson and soliton stars. No doubt in the study of the evolution of such massive stars the "stability factor" is crucial as the stars with $M >$ 7–8 $M_\odot$ are expected to be instable (see, for example, Ellis et al. [14] and Chandrasekar [31]). However, the detailed study of this factor with reference to an ESD star is still desirable.

In other words, the evolution of the massive stars is expected to follow a somewhat different theoretical framework than that of the less massive ones. As a consequence, the ESD matter either in the form of a star or stars with ESD and/or quark matter in the core may play an important role in: (i) a would-be supernova and/or the process of black hole formation; (ii) the collapse of the infalling stellar matter due to condensation of ESD in the diquark–gluon plasma as speculated by Frederiksson [25]; and (iii) understanding the early phases of the origin of the universe. Furthermore, for these superdense and massive systems having diquarks as constituents, the study of the phenomena of Bose–Einstein condensations [3], colour superconductivity [3] and the phase transitions with reference to the chiral symmetry [36] would raise very interesting possibilities. While these speculations have been around in the literature for some time, more precise studies in this regard, particularly within the present framework, are desirable and we hope to address some of them in the future.

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